PHYS 4073 - Quantum Mechanics- Test 1 - Fall 2010

All problems are worth 25 points. Turn in solutions to four of the six problems to be graded. If you turn in more than four solutions, I will grade the first four. You are allowed to drop one-half of a test, so I will take the first two problems turned in as the first half-test and the second two problems turned in as the second half-test.

1 A particle is in the initial state $\psi(x,0) = A\exp(-ax)$ for $x > 0$ and $\psi = 0$ for $x < 0$. Compute the probability of finding the particle in the region $0 < x < 1/a$.

2 The initial state of a particle of mass m in an infinite square well is $\psi(x,0) = \frac{1}{\sqrt{2}}\phi_1(x) - \frac{1}{\sqrt{2}}\phi_2(x)$ where $\phi_n = \sqrt{\frac{2}{a}}\sin(n\pi x/a)$. Note, this initial state is already normalized. Compute the wave function as a function of time, $\psi(x,t)$. Compute the expectation value of x, $< x >$, as a function of time.

3 The initial state of a particle in an infinite square well that extends from 0 to a is $\psi(x,0) = A\sin(\pi x/a)$ for $0 < x < a/2$ and zero otherwise. Compute the probability of finding the particle in the ground state and the probability of finding the particle in the first excited state.

4 The wavefunction of a particle of mass m is $\psi(x,0) = A\sin(\pi x/b)$ for $0 < x < b$ and zero otherwise. Check the uncertainty relation for this state.

5 A particle is directed at a finite step potential where

$$V(x) = \begin{cases} 0 & \text{if } x < 0 \\ V_0 & \text{if } 0 < x \end{cases}$$

Calculate the reflection coefficient from the time independent Schrodinger equation showing all steps for the $E > V_0$ case.

6 Model the hydrogen atom as a finite one-dimensional well of width $a = 1 \times 10^{-10}$m, two Bohr radii. The ground-state energy of hydrogen is $E_0 = 13.6$eV with $1\text{eV} = 1.602 \times 10^{-19}$J. Using the Bohr model, this means the potential energy of hydrogen, $V = -V_0 = -27.2$eV. Compute the lowest energy bound state for the electron in this well. How many total bound states does the well have? Note you will not need E_0, that was just me thinking aloud.