Homework 8

Due Friday 4/4/2003 at beginning of class

Total 60

Fowles Problems

- 5 • 8.2
- 10 • 8.6
- 5 • 8.8
- 10 • 8.9 (Part a 8, Part 2)
- 5 • 8.12
- 10 • 8.16
- 10 • 8.20
- 5 • 8.22

X **Bonus Problem B.1** What is the name of the geometric figure inside Jennifer's Magic Eight Ball. You may make an observation if you wish, but you may not take it apart.

icosahedrom

where k_{cm} is the radius of gyration of the body about its center of mass. In this case the point O is the instantaneous center of rotation of the body just after impact. O' is called the *center of percussion* about O. The two points are related in the same way as the centers of oscillation, defined previously in our analysis of the physical pendulum (Equation 8.4.13).

Anyone who has played baseball knows that if the ball hits the bat in just the right spot there will be no "sting" on impact. This "right spot" is just the center of percussion about the point at which the bat is held.

PROBLEMS

8.1 Find the center of mass of each of the following:

(a) A thin wire bent into the form of a three-sided, block-shaped "⊔" with each segment of equal length b

(b) A quadrant of a uniform circular lamina of radius b

(c) The area bounded by parabola $y = x^2/b$ and the line $y = b$

(d) The volume bounded by paraboloid of revolution $z = (x^2 + y^2)/b$ and the plane $z = b$

(e) A solid uniform right circular cone of height b

8.2 The linear density of a thin rod is given by $\rho = cx$, where c is a constant and x is the distance measured from one end. If the rod is of length b, find the center of mass.

8.3 A solid uniform sphere of radius a has a spherical cavity of radius $a/2$ centered at a point $a/2$ from the center of the sphere. Find the center of mass.

8.4 Find the moments of inertia of each of the objects in Problem 8.1 about their symmetry axes.

8.5 Find the moment of inertia of the sphere in Problem 8.3 about an axis passing through the center of the sphere and the center of the cavity.

8.6 Show that the moment of inertia of a solid uniform octant of a sphere of radius a is $(2/5)ma^2$ about an axis along one of the straight edges. (*Note:* This is the same formula as that for a solid sphere of the same radius.)

8.7 Show that the moments of inertia of a solid uniform rectangular parallelepiped, elliptic cylinder, and ellipsoid are, respectively, $(m/3)(a^2 + b^2)$, $(m/4)(a^2 + b^2)$, and $(m/5)(a^2 + b^2)$, where m is the mass, and $2a$ and $2b$ are the principal diameters of the solid at right angles to the axis of rotation, the axis being through the center in each case.

8.8 Show that the period of a physical pendulum is equal to $2\pi(d/g)^{1/2}$, where d is the distance between the point of suspension O and the center of oscillation O'.

8.9 **(a)** An idealized simple pendulum consists of a particle of mass M suspended by a thin massless rod of length a. Assume that an actual simple pendulum consists of a thin rod of mass m attached to a spherical bob of mass $M - m$. If the radius of the spherical bob is equal to b, and the length of the thin rod is equal to $a - b$, calculate the ratio of the period of the actual simple pendulum to the idealized simple one.

(b) Calculate a value for this ratio if $m = 10$ g, $M = 1$ kg, $a = 1.27$ m, and $b = 5$ cm.

8.10 The period of a physical pendulum is 2 s. (Such a pendulum is called a "seconds" pendulum.) The mass of the pendulum is M, and its center of mass is 1 m below the axis of oscillation. A particle of mass m is attached to the bottom of the pendulum, 1.5 m below the axis, in line with the center of gravity. It is then found that the pendulum "loses" time at the rate of 20 s/day. Find the ratio of m to M.

8.11 A circular hoop of radius a swings as a physical pendulum about a point on the circumference. Find the period of oscillation for small amplitude if the axis of rotation is

(a) Normal to the plane of the hoop

(b) In the plane of the hoop

8.12 A uniform solid ball has a few turns of light string wound around it. If the end of the string is held steady and the ball is allowed to fall under gravity, what is the acceleration of the center of the ball? (Assume the string remains vertical.)

8.13 Two people are holding the ends of a uniform plank of length l and mass m. Show that if one person suddenly lets go, the load supported by the other person suddenly drops from $mg/2$ to $mg/4$. Show also that the initial downward acceleration of the free end is $\frac{3}{2}g$.

8.14 A uniform solid ball contains a hollow spherical cavity at its center, the radius of the cavity being $\frac{1}{2}$ the radius of the ball. Show that the acceleration of the ball rolling down a rough inclined plane is just 98/101 of that of a uniform solid ball with no cavity. (*Note:* This suggests a method for nondestructive testing.)

8.15 Two weights of mass m_1 and m_2 are tied to the ends of a light inextensible cord. The cord passes over a rough pulley of radius a and moment of inertia I. Find the accelerations of the weights, assuming $m_1 > m_2$ and ignoring friction in the axle of the pulley.

8.16 A uniform right-circular cylinder of radius a is balanced on the top of a perfectly rough fixed cylinder of radius b ($b > a$), the axes of the two cylinders being parallel. If the balance is slightly disturbed, show that the rolling cylinder leaves the fixed one when the line of centers makes an angle with the vertical of $\cos^{-1}(4/7)$.

8.17 A uniform ladder leans against a smooth vertical wall. If the floor is also smooth, and the initial angle between the floor and the ladder is θ_0, show that the ladder, in sliding down, will lose contact with the wall when the angle between the floor and the ladder is $\sin^{-1}(\frac{2}{3}\sin\theta_0)$.

8.18 At Cape Canaveral a Saturn V rocket stands in a vertical position ready for launch. Unfortunately, before firing, a slight disturbance causes the rocket to fall over. Find the horizontal and vertical components of the reaction on the launch pad as functions of the angle θ between the rocket and the vertical at any instant. Show from this that the rocket will tend to slide backward for $\theta < \cos^{-1}(2/3)$ and forward for $\theta > \cos^{-1}(2/3)$. (Assume the rocket to be a thin uniform rod.)

8.19 A ball is initially projected, without rotation, at a speed v_0 up a rough inclined plane of inclination θ and coefficient of sliding friction μ_k. Find the position of the ball as a function of time, and determine the position of the ball when pure rolling begins. Assume that μ_k is greater than $\frac{2}{7}\tan\theta$.

8.20 A billiard ball of radius a is initially spinning about a horizontal axis with angular speed ω_0 and with zero forward speed. If the coefficient of sliding friction between the ball and the billiard table is μ_k, find the distance the ball travels before slipping ceases to occur.

8.21 Figure P8.21 illustrates two discs of radii a and b mounted inside a fixed, immovable circular track of radius c, such that $c = a - 2b$. The central disc A is mounted to a drive axle at point O. Disc B is sandwiched between disc A and track C and can roll without slipping when disc A is driven by an externally applied torque through its drive axle. Initially, the system is at rest such that the dashed lines denoting the spatial orientation of discs A and B line up horizontally in the figure. A constant torque K is applied for a time t_0 through the drive axle causing disc A to rotate, such that at time t_0 the

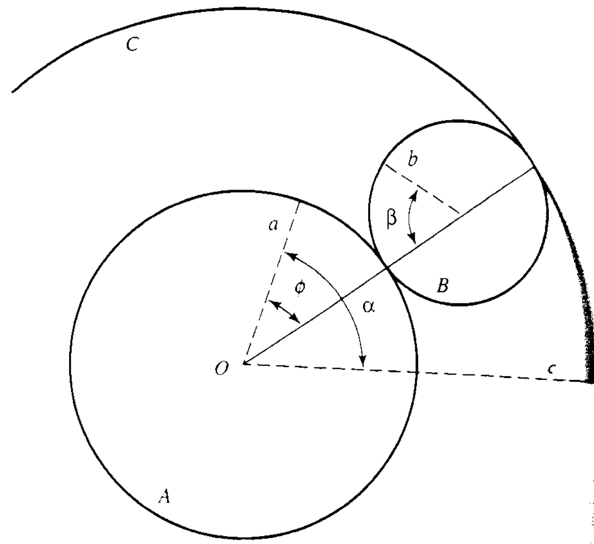


Figure P8.21

dashed line denoting its spatial orientation makes an angle α with the horizontal. Disc B rolls between the track and disc A, and its orientation is denoted by the dashed line making and angle β with the direction toward O. Calculate the final angular speeds of the two discs, ω_A and ω_B.

8.22 A thin uniform plank of length l lies at rest on a horizontal sheet of ice. If the plank is given a kick at one end in a direction normal to the plank, show that the plank will begin to rotate about a point located a distance $l/6$ from the center.

8.23 Show that the edge (cushion) of a billiard table should be at a height of 7/10 of the diameter of the billiard ball in order that no reaction occurs between the table surface and the ball when the ball strikes the cushion.

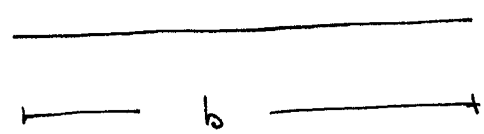
8.24 A ballistic pendulum is made of a long plank of length l and mass m. It is free to swing about one end O and is initially at rest in a vertical position. A bullet of mass m' is fired horizontally into the pendulum at a distance l' below O, the bullet coming to rest in the plank. If the resulting amplitude of oscillation of the pendulum is θ_0, find the speed of the bullet.

8.25 Two uniform rods AB and BC of equal mass m and equal length l are smoothly joined at B. The system is initially at rest on a smooth horizontal surface, the points A, B, and C lying in a straight line. If an impulse $\boldsymbol{P}$ is applied at A at right angles to the rod, find the initial motion of the system. (*Hint:* Isolate the rods.)

COMPUTER PROBLEMS

C 8.1 The table shown here displays the density of a 10-solar mass star versus radial distance from its core. The densities are given as $\log_{10} \rho/\rho_c$, where ρ is the density at the distance r and ρ_c is the core density of the star. The distances are given as fractions of the star's radius ($r/R_\star$), where $R_\star$ is equal to 4 solar radii.

8.2

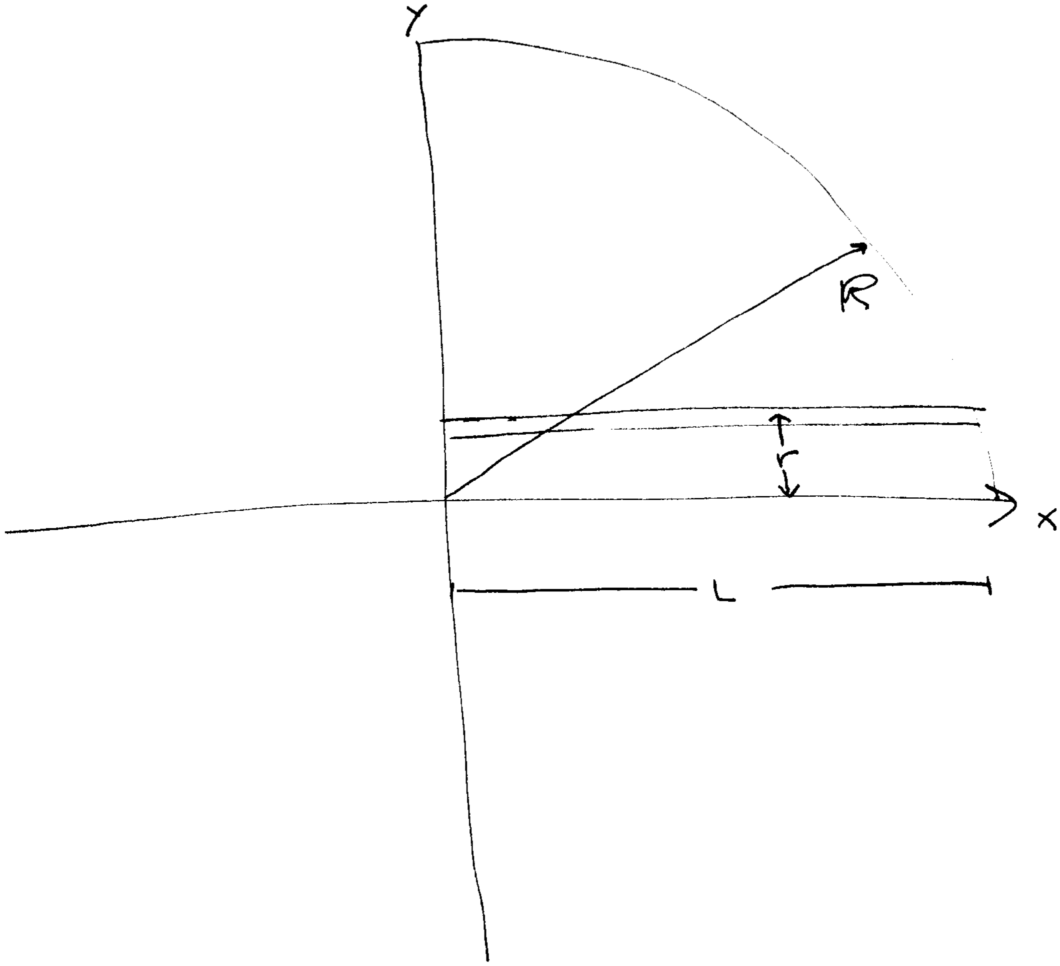


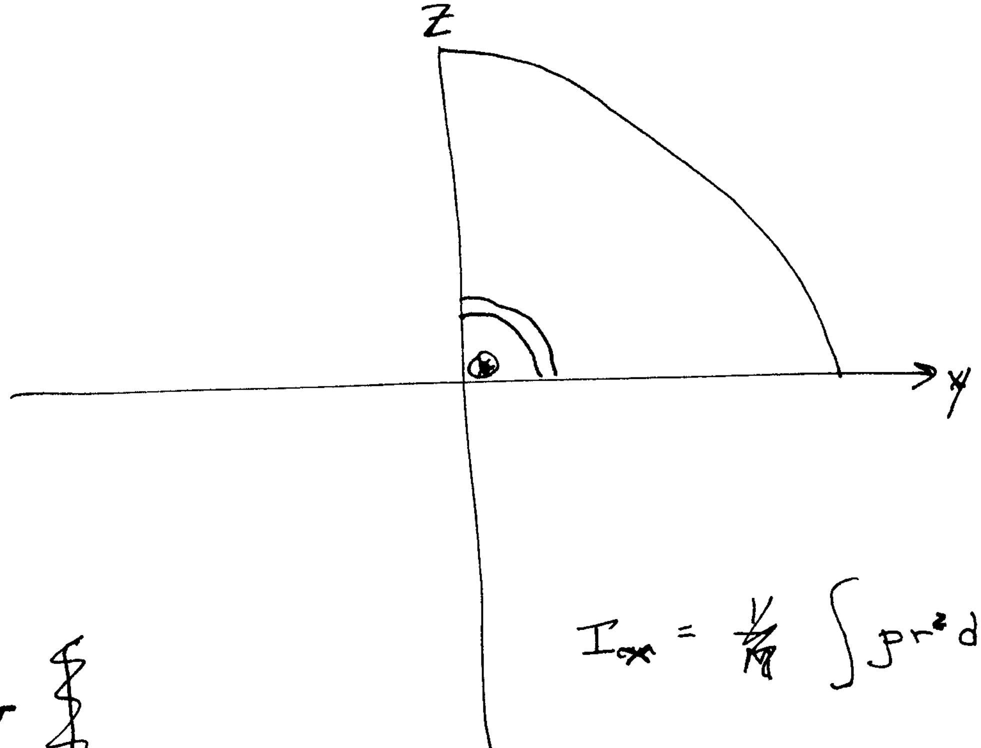
$\lambda = cx$

$$r_{cm} = \frac{\int_0^b x\, dm}{\int_0^b dm} = \frac{\int_0^b cx^2 dx}{\int_0^b cx\, dx}$$

$$= \frac{\frac{b^3}{3}}{\frac{b^2}{2}} = \frac{2}{3} b$$

8.6





$I_{cm} = \int_0^R dr$

$I_{cm} = \cancel{\tfrac{1}{M}} \int \rho r^2 \, dV$

Divide the solid into thin cylindrical quarter shells. The volume of a shell is

$$dV = L\left(\frac{2\pi r}{4}\right) dr$$

$$L = \sqrt{R^2 - r^2}$$

$$I_x = \frac{\pi \rho}{2}\int_0^R dr\, r^3 \sqrt{R^2 - r^2}$$

$$u = R^2 - r^2 \qquad du = -2r\,dr$$

$$I_x = -\frac{\pi \rho}{4}\int_0^R u^{1/2}\, du = -\frac{\pi \rho}{4}\left(\frac{u^{3/2}}{3/2}\right)_{0(0)}^{R(R)}$$

$$= \frac{-\pi \rho}{6}\left[0 - R^{3/2}\right]$$

$$= \frac{\pi \rho R^{3/2}}{6}$$

$$M = \frac{1}{8}\frac{4}{3}\rho \pi R^3$$

$$= \frac{1}{6}\rho \pi R^3$$

$$\frac{I_x}{M} = R^{-3/2}$$

From integral table,

$$\int dx\, x^3 \sqrt{a^2-x^2} = \frac{(a^2-x^2)^{5/2}}{5} - \frac{a^2(a^2-x^2)^{3/2}}{3}$$

$$\int_0^R dr\, r^3 \sqrt{R^2-x^2} = \left[\frac{-R^5}{5} - \frac{R^5}{3}\right]$$

$$= \frac{2}{15} R^5$$

$$I_x = \frac{\pi\rho}{2}\,\frac{2}{15}R^5 = \frac{\pi}{15}R^5$$

$$\frac{I_x}{M} = \frac{\rho\frac{\pi}{15}R^5}{\frac{1}{8}\left(\frac{4}{3}\right)\pi\rho R^3} = \frac{2}{5}R^2$$

(8.8)

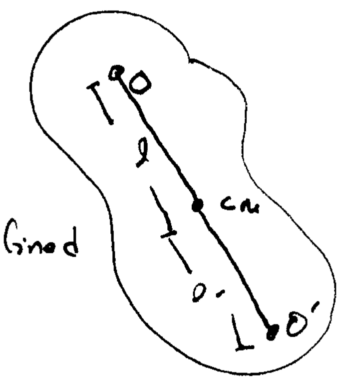
Period of a Physical Pendulum

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

I = moment of inertia about pivot.

l = Distance from pivot to CM

$d \equiv l + l'$



$O' \equiv$ Center of oscillation - Defined by $l l' = k_{cm}^2$

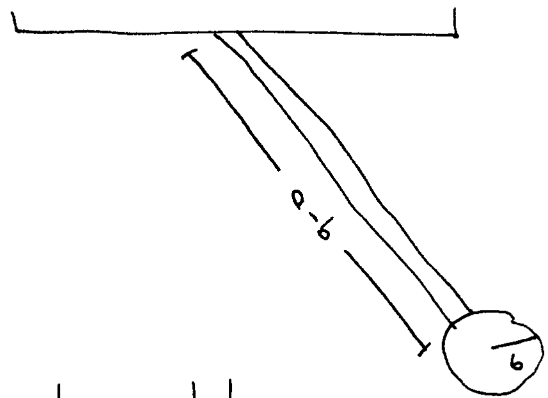
Parallel Axis Thm $\quad I = I_{cm} + ml^2$

$$= m k_{cm}^2 + ml^2$$

$$T = 2\pi\sqrt{\frac{k_{cm}^2 + l^2}{gl}} = 2\pi\sqrt{\frac{l l' + l^2}{gl}}$$

$$= 2\pi\sqrt{\frac{l' + l}{g}} = 2\pi\sqrt{d/g}$$

8.9



The period of a physical pendulum is

$$T_R = 2\pi \sqrt{\frac{I}{Mg\,l}}$$

$l \equiv$ distance from point of support to CM.

The period of an idealized pendulum is

$$T_0 = 2\pi \sqrt{\frac{l}{g}}$$

The ratio of the periods is

$$T_R / T_0 = \sqrt{\frac{I}{M\,l_{cm}}}$$

<u>Compute I</u>

$$I = I_{rod} + I_{ball}$$

~~I_{rod}~~

$$I_{rod} = \frac{m(a-b)^2}{3} \qquad I_{ball} = (M-m)\,a^2 + (M-m)\frac{2}{5}b^2$$

$$I = \frac{m(a-b)^2}{3} + (M-m)a^2 + \frac{2}{5}(M-m)b^2$$

$$= \frac{ma^2}{3} - \frac{2mab}{3} + \frac{mb^2}{3} + Ma^2 - ma^2$$

$$+ \frac{2}{5}Mb^2 - \frac{2}{5}mb^2$$

$$= \left(\frac{m}{3} + M - m\right)a^2 - \frac{2mab}{3} + \left(\frac{mb^2}{3} + \frac{2}{5}M - \frac{2}{5}m\right)b^2$$

$$= \left(M - \frac{2m}{3}\right)a^2 - \frac{2mab}{3} + \left(\frac{2}{5}M - \frac{1}{15}m\right)b^2$$

$$\ell_{cm} = \frac{1}{M}\left(m\, r_{rod}^{cm} + (M-m)\, r_{ball}^{cm}\right)$$

$$= \frac{1}{M}\left(m\frac{(a-b)}{2} + (M-m)a\right)$$

$$= \frac{1}{M}\left(\frac{ma}{2} - \frac{mb}{2} + Ma - ma\right)$$

$$= \frac{1}{M}\left(Ma - \frac{m}{2}(a+b)\right)$$

$$\frac{T_R}{T_0} = \frac{2\pi\sqrt{\dfrac{I}{M g l_{cm}}}}{2\pi\sqrt{\dfrac{M a^2}{M g a}}} = \sqrt{\frac{I}{M l_{cm} a}}$$

$$= \sqrt{\frac{\left(M - \frac{2}{3} m\right) a^2 - \frac{2mab}{3} + \left(\frac{2}{5} M - \frac{1}{5} m\right) b^2}{\cancel{M} a \left(\frac{1}{\cancel{M}}\right)\left(Ma - \frac{m}{2}(a+b)\right)}}$$

~~(b)~~

—OR—

$$\frac{T_R}{T_0} = \sqrt{\frac{\frac{m}{3}(a-b)^2 + (M-m)\left(a^2 + \frac{2}{5} b^2\right)}{a\left(Ma - m/2\,(a+b)\right)}}$$

$\alpha = m/M \qquad \beta = b/a$ (Not required

$$\frac{T_R}{T_0} = \sqrt{\frac{\frac{\alpha}{3}(1-\beta)^2 + (1-\alpha)\left(1 + \frac{2}{5}\beta^2\right)}{1 - \frac{\alpha}{2}(1+\beta)}}$$

$$\alpha = \frac{m}{M} = \frac{0.01\,\text{kg}}{1\,\text{kg}} = 0.01 \qquad B = b/a = \frac{0.05\,\text{m}}{1.27\,\text{m}} = 0.039$$

$$\left(\frac{T_R}{T_0}\right)^2 = \frac{\frac{0.01}{3}\left(1-0.039\right)^2 + \left(1-0.01\right)\left(1+\frac{2}{5}0.039^2\right)}{1-\frac{0.01}{2}\left(1+0.039\right)}$$

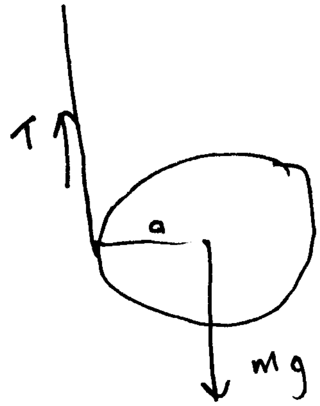
$$= \frac{0.003076 + 0.99060}{0.994805}$$

$$= 0.99887$$

$$\frac{T_R}{T_0} = 0.9994$$

(Anything near 0.999 ... accepted)

8.12



<u>Energy</u> $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgh = \text{constant}$

$v = a\omega \qquad h = \cancel{2\pi}\, a\,\theta$

$$\frac{1}{2}ma^2\omega^2 + \frac{1}{2}I\omega^2 + mga\theta = \text{constant}$$

<u>Force</u> $T - mg = ma_{cm} = ma\dot{\omega}$

<u>Torque</u> $Ta = -I\dot{\omega}$

Torque is negative because causes clockwise rotation, $\dot{\omega} < 0$

$$-\frac{I\dot{\omega}}{a} - mg = ma\dot{\omega}$$

$$\cancel{\dot{\omega} \neq \frac{I}{a}} \left(ma + \frac{I}{a}\right)\dot{\omega} = -mg$$

$$\dot{\omega} = \frac{-mg}{ma + \frac{I}{a}}$$

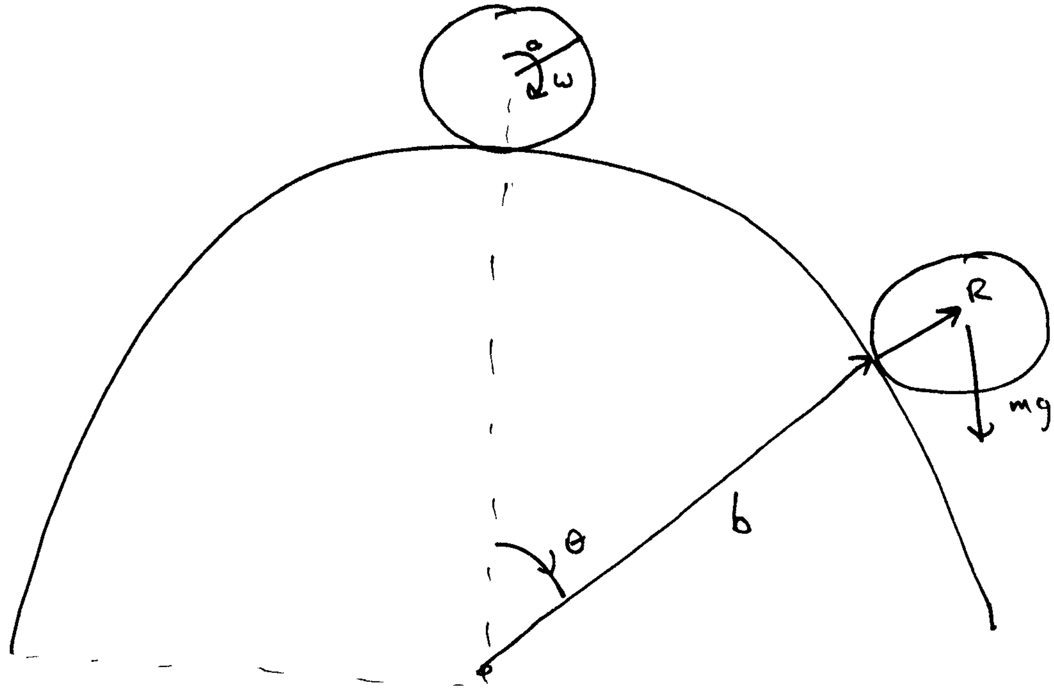
$$a_{cm} = a\dot{\omega} = \frac{-mga}{ma + I/a^2}$$

$$= \frac{-g}{1 + \frac{I}{ma^2}}$$

For a solid sphere, $I_{cm} = \frac{2}{5} ma^2$

$$a_{cm} = \frac{-g}{1 + 2/5} = \frac{-5g}{7}$$

8.16



For rolling without slipping $\quad \omega = \dfrac{v_{cm}}{a}$

At top all energy is potential, conserve energy

$$mg(a+b) = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mg(a+b)\cos\theta$$

$$I = \frac{1}{2}ma^2$$

$$mg(a+b) = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}ma^2\right)\left(\frac{v}{a}\right)^2 + mg(a+b)\cos\theta$$

$$= \frac{1}{2}mv^2\left[1 + \frac{1}{2}\right] + mg(a+b)\cos\theta$$

Radial
Force Balance (inward positive)

$$mg\cos\theta - R = \frac{mv^2}{a+b}$$

The cylinder leaves the surface when $R = 0$.

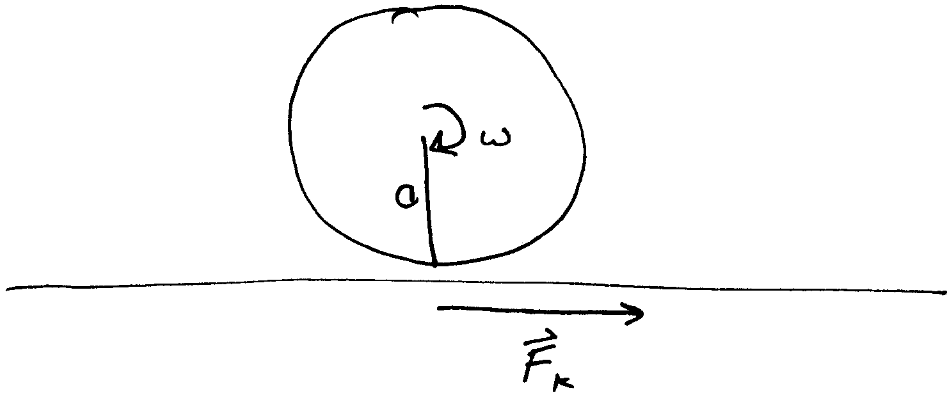
$$g(a+b)\cos\theta = v^2$$

$$mg(a+b) = \frac{3}{4} mg(a+b)\cos\theta + mg(a+b)\cos\theta$$

$$1 = \left(\frac{3}{4} + 1\right)\cos\theta = \frac{7}{4}\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{4}{7}\right)$$

8.20



The only force (moving) on the ball is the force of kinetic friction, $F_k = \mu_k mg$

So the ball moves with a constant acceleration,

$$a_{cm} = \mu_k g$$

$$x(t) = \frac{1}{2} a_{cm} t^2 = \frac{\mu_k g}{2} t^2$$

$$v(t) = a_{cm} t = \mu_k g t$$

The force also exerts a torque that slows the balls rotation rate.

$$N = -Fa = I \dot{\omega}$$

So there is also a constant angular acceleration.

$$\dot{\omega} = -\frac{F a}{I} \qquad I = \frac{2}{5} m a^2$$

$$\dot{\omega} = -\frac{m g \mu_k a}{\frac{2}{5} m a^2} = -\frac{5}{2} \frac{g \mu_k}{a}$$

$$\omega(t) = \omega_0 - \frac{5}{2} \frac{g \mu_k}{a} t$$

The ball stops sliding when the condition of rolling is met,

$$\omega(t) = \frac{v(t)}{a}$$

$$\omega_0 - \frac{5}{2} \frac{g \mu_k}{a} t = \frac{\mu_k g}{a} t$$

$$\omega_0 = \frac{\mu_k g}{a} \left(1 + 5/2 \right) t = 7/2 \frac{\mu_k g}{a} t$$

$$t = \frac{2}{7} \frac{a \omega_0}{\mu_k g}$$

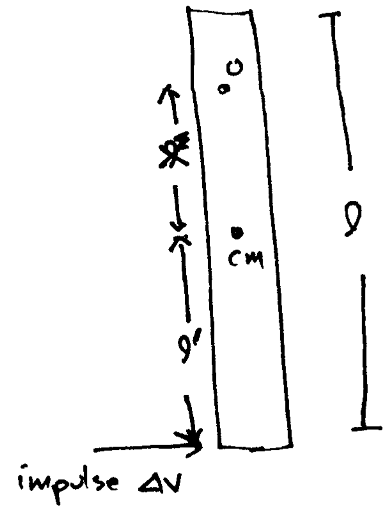
Distance Travelled before Slipping Stops

$$d = \frac{\mu_k g}{2} t^2$$

$$= \left(\frac{\mu_k g}{2}\right)\left(\frac{2}{7}\,\frac{a\omega_0}{\mu_k g}\right)^2$$

$$= \frac{2}{49}\,\frac{a^2\omega_0^2}{\mu_k g}$$

8.22



The instantaneous axis of rotation, is the point with zero velocity immediately after impact.

From eqn, 8.7.9

$$V_o = v_{cm} - \omega l = P\left(\frac{1}{M} - \frac{x l'}{I_{cm}}\right) = 0$$

$$x l' = k_{cm}^2 = \frac{l^2}{12}$$

$$l' = l/2$$

$$\boxed{x = l/6}$$